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Optimization of Thermally and Geometrically Asymmetric Trapezoidal Fins

H. S. Kang*
Kangwon National University, Chuncheon 200-701, Republic of Korea
and
D. C. Look, Jr.†
University of Missouri–Rolla, Rolla, Missouri 65409-0050

Presented is an analysis of thermally and geometrically asymmetric trapezoidal fins using a two-dimensional analytical method. Some isothermal lines within this thermally and geometrically asymmetric fin are shown. The optimum heat loss is chosen to be 98% of the maximum heat loss by comparison of the increasing rate of dimensionless fin length and that of heat loss from the fin. The optimum heat loss, the corresponding dimensionless fin length, and fin volume are presented as a function of the ratio of the fin bottom-surface to top-surface Biot numbers, the ratio of the fin tip-surface to top-surface Biot numbers, and a fin shape factor. Results show that the effect of fin shape on the optimum heat loss is very small while the optimum heat loss increases linearly as the ratio of fin bottom-surface to top-surface Biot numbers increases.

Nomenclature

Bi_i = Biot number of the ith fin surface, $h_i \ell / k$
c = fraction of heat transfer to the maximum possible value $(0 \le c \le 1)$
h_i = heat-transfer coefficient of ith surface, W/m^2-K
k = thermal conductivity of fin material, W/m-K
L' = fin length, m; dimensionless form, $L = L'/\ell$
ℓ = fin height at the base, m
Q^* = dimensionless optimum heat transfer from the fin (or 98% of $Q_{\max}$)
q = heat transfer from the fin, W; dimensionless form, $Q = q/(k \cdot \theta_b)$
s = slope of fin's top surface, $0 < (1-\xi)/L < 1/L$
T = temperature within the fin, K
V' = fin volume for a unit width, m^3; dimensionless form, V'/ℓ^3
x' = horizontal coordinate, m; dimensionless form, $x = x'/\ell$
y' = vertical coordinate, m; dimensionless form, $y = y'/\ell$
α = ratio of fin bottom-surface to top-surface Biot numbers, Bi_2/Bi_1
β = ratio of fin tip-surface Biot number to fin top-surface Biot number, Bi_3/Bi_1
θ = dimensionless temperature, $(T - T_\infty)/(T_w - T_\infty)$
θ_b = modified fin base temperature $(T_w - T_\infty)$, K
λ_n = eigenvalues, $n = 1, 2, 3, \ldots$
ξ = fin shape factor ($\xi = 1$ for rectangular fin, $\xi = 0$ for triangular fin)

Subscripts

b = fin base
max = maximum
w = wall
1 = fin top
$1r$ = one-dimensional analysis for rectangular fin
2 = fin bottom
$2r$ = two-dimensional analysis for rectangular fin
3 = fin tip
∞ = surrounding

Superscript

$*$ = optimum

Introduction

EXTENDED surfaces as heat-transfer enhancement devices have been used in many engineering applications. Optimization procedures in heat-transfer problems, especially fin problems, has been an important concern for a long time. One of the usual optimization procedures is to fix a suitable simple profile and then determine the dimensions of the fin to yield maximum heat dissipation for a given fin volume or mass.

The studies of the optimum dimensions for various fin shapes yielded several publications.[1–7] Brown[1] considered the annular rectangular profile, and Hrymak et al.[2] investigated an efficient numerical method to discover the optimum shape for a fin subjected to both convective and radiative heat loss. Ullmann and Kalman[3] analyzed four different annular profiles (rectangular, triangular, parabolic, and hyperbolic) and found that the parabolic fin has the best performance of the four examined fin shapes. Also, Razelos and Imre[4] investigated rectangular, triangular, and trapezoidal annular fins with variable thermal parameters and showed that the optimum base thickness and volume of the fin are inversely proportional to the thermal conductivity if the thermal conductivity is constant. Gerencser and Razan[5] studied an optimum pin fin array of variable cross section for a given fin material per unit base area and showed that the effectiveness of the array increases rapidly for a fixed volume per unit area as the fin spacing decreases. Recently, Casarosa and Franco[6] approached the optimum design of single longitudinal fins with constant thickness considering different uniform heat-transfer coefficients on the fin surfaces, and Laor and Kalman[7] examined longitudinal, spine, and annular fins each with rectangular, triangular, and parabolic shapes and showed the efficiency and optimum dimensions graphically to correlate to a simple equation that will lead to a quick and accurate design. All of these optimization studies were done using a one-dimensional analysis. However, a two-dimensional analysis can show the temperature distribution along

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*Associate Professor, Department of Mechanical Engineering.
†Professor Emeritus, Department of Mechanical and Aerospace Engineering and Engineering Mechanics. Associate Fellow AIAA.

the fin height, and it can be applied to a wide range of physical variables with more accuracy (i.e., for a fin which is relatively thick or for a fin under asymmetric conditions). Also, two-dimensional studies of annular fins have been published. For example, Look[8] discussed the heat loss ratio between a fin and a bare pipe for a radial fin of uniform thickness on a pipe using both one- and two-dimensional methods, and Kundu and Das[9] presented a comparison between the performance of concentric and eccentric fins with various radius ratios and determined the optimum dimensions for eccentric annular fins using a semi-analytical method. Although these papers assume that the convection coefficients are equal, Look and Kang[10] discussed the optimization of a straight rectangular fin under thermally asymmetric conditions.

The purpose of this study is to present the optimization for thermally and geometrically asymmetric straight trapezoidal profile fins using a two-dimensional analytic method when the fin base dimension is fixed. The shape of this geometrically asymmetric trapezoidal fin can be changed from a geometrically asymmetric triangular to rectangular fin by adjusting a fin shape factor. The analysis is based upon the usual assumptions[11] (i.e., constant properties, steady state, no heat source, and Newton's law is valid).

Analysis

Two-Dimensional Thermally and Geometrically Asymmetric Trapezoidal Fin

The nondimensional form of the governing two-dimensional differential equation for a thermally and geometrically asymmetric trapezoidal fin, as illustrated in Fig. 1, can be written as Eq. (1):

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0 \qquad (1)$$

Three corresponding two-dimensional boundary conditions and one energy balance equation are required to define the problem as shown in Eqs. (2–5):

$$\theta|_{x=0} = 1, \qquad 0 \le y \le 1 \qquad (2)$$

$$\left.\frac{\partial\theta}{\partial y}\right|_{y=0} - Bi_2 \cdot \theta|_{y=0} = 0, \qquad 0 \le x \le L \qquad (3)$$

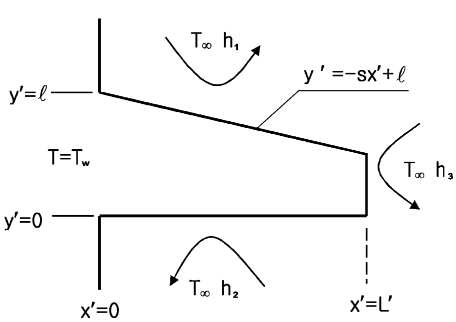

Fig. 1a Thermally and geometrically asymmetric trapezoidal fin schematic.

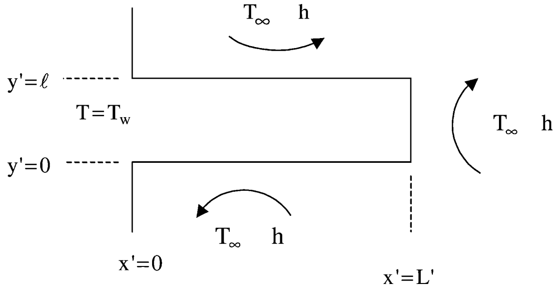

Fig. 1b Geometry of a rectangular fin.

$$\left.\frac{\partial\theta}{\partial x}\right|_{x=L} + Bi_3 \cdot \theta|_{x=L} = 0, \qquad 0 \le y \le (1 - s \cdot L) \qquad (4)$$

and

$$-\int_0^1 \left.\frac{\partial\theta}{\partial x}\right|_{x=0} \mathrm{d}y = Bi_1 \cdot \sqrt{1 + \left(\frac{1}{s}\right)^2} \cdot \int_{1-s\cdot L}^{1} \theta\,\mathrm{d}y$$
$$- \int_0^{1-s\cdot L} \left.\frac{\partial\theta}{\partial x}\right|_{x=L} \mathrm{d}y + \int_0^L \left.\frac{\partial\theta}{\partial y}\right|_{y=0} \mathrm{d}x \qquad (5)$$

The energy balance Eq. (5) means physically that the heat transfer through a fin base by conduction equals the sum of the heat loss from all of the fin surfaces by convection. Because surrounding Biot numbers can be different, the top-surface Biot number is arbitrarily chosen as a base value and denoted by Bi_1. In the following discussion the optimum dimensions are presented as a function of α and β. The solution for the temperature distribution $\theta\ (x, y)$ within the fin, obtained by solving Eq. (1) with Eqs. (2–4) using the separation of variables method, is

$$\theta = \sum_{n=1}^{\infty} N_n \cdot f(x) \cdot f(y) \qquad (6)$$

where

$$f(x) = \cosh(\lambda_n x) - f_n \cdot \sinh(\lambda_n x) \qquad (7)$$

$$f(y) = \cos(\lambda_n y) + g_n \cdot \sin(\lambda_n y) \qquad (8)$$

$$f_n = \frac{[\lambda_n \cdot \sinh(\lambda_n L) + Bi_3 \cdot \cosh(\lambda_n L)]}{[\lambda_n \cdot \cosh(\lambda_n L) + Bi_3 \cdot \sinh(\lambda_n L)]} \qquad (9)$$

$$g_n = \frac{Bi_2}{\lambda_n} \qquad (10)$$

$$N_n = \frac{A_n}{(B_n + C_n + D_n)} \qquad (11)$$

$$A_n = 4\{\lambda_n \cdot \sin(\lambda_n) + Bi_2 \cdot [1 - \cos(\lambda_n)]\} \qquad (12)$$

$$B_n = 2\lambda_n^2 + \lambda_n \cdot \sin(2\lambda_n) \qquad (13)$$

$$C_n = 2Bi_2 \cdot [1 - \cos(2\lambda_n)] \qquad (14)$$

and

$$D_n = Bi_2^2 \cdot [2 - \sin(2\lambda_n)/\lambda_n] \qquad (15)$$

The eigenvalues λ_n are obtained using an energy balance equation, which was derived from Eq. (5). This equation is complicated and is skipped here. From this equation the first eigenvalue is obtained using an incremental search method, and the rest of the eigenvalues are obtained by a forced analytic method.[12] The equation for the heat loss from this thermally and geometrically asymmetric trapezoidal fin per unit width is given by Eq. (16):

$$q = \int_0^{\ell} -k \left.\frac{\partial T}{\partial x'}\right|_{x'=0} \mathrm{d}y' \qquad (16)$$

The resulting dimensionless form is calculated using Eq. (17):

$$Q = \frac{q}{(k\theta_b)} = \sum_{n=1}^{\infty} N_n \cdot f_n \cdot \{\sin(\lambda_n) + g_n \cdot [1 - \cos(\lambda_n)]\} \qquad (17)$$

The dimensionless fin volume for a unit width is obtained using Eq. (18):

$$V = \frac{V'}{\ell^3} = \frac{\left(\int_0^{L'} y'\,\mathrm{d}x'\right)}{\ell^3} = \frac{L \cdot (1 + \xi)}{2} \qquad (18)$$

The optimum heat-loss condition is

$$\frac{\mathrm{d}Q}{\mathrm{d}L} = 0 \tag{19}$$

For the special case of a rectangular profile fin, the result of this optimization is

$$\frac{\sum_{n=1}^{\infty}\left(Bi_3^2 - \lambda_n^2\right)\cdot \sec h^2(\lambda_n L)}{[\lambda_n + Bi_3 \cdot \tanh(\lambda_n L)]} = 0 \tag{20}$$

Equation (20) will be satisfied as L becomes large; then $f_n \to 1$. Thus, the limiting value of heat loss from the fin is

$$Q_{\text{limiting}} = \sum_{n=1}^{\infty} N_n \cdot \{\sin(\lambda_n) + g_n \cdot [1 - \cos(\lambda_n)]\} \tag{21}$$

This limiting value will be maximum when the fin's Biot numbers are small; it might not be the maximum when the fin's surrounding Biot numbers are large. This is because the heat loss from the fin is less than that from the bare wall when the fin's Biot numbers are large (for example, all Biot numbers are over 1). So the maximum heat loss from the rectangular fin under our usual circumstance can be expressed by Eq. (22):

$$Q_{\max} = \sum_{n=1}^{\infty} N_n \cdot \{\sin(\lambda_n) + g_n \cdot [1 - \cos(\lambda_n)]\} \tag{22}$$

Two-Dimensional Rectangular Fin

To compare the analysis for thermally and geometrically asymmetric trapezoidal fin, one- and two-dimensional analyses for a retangular fin, as shown in Fig. 1b, will be quickly made here. For the two-dimensional analysis the governing differential equation and three boundary conditions are the same as the preceding analysis, except $Bi_2 = Bi_3 = Bi$, and energy balance [Eq. (5)] is replaced as top boundary Eq. (23):

$$\left.\frac{\partial\theta}{\partial y}\right|_{y=1} + Bi \cdot \theta|_{y=1} = 0, \qquad 0 \le x \le L \tag{23}$$

The solutions for temperature and heat loss are of the same form as the preceding analysis except $Bi_2 = Bi_3 = Bi$, and eigenvalues are obtained from Eq. (24), which is based on Eq. (23):

$$Bi^2 \sin(\lambda_n) + 2\lambda_n Bi \cos(\lambda_n) - \lambda_n^2 \sin(\lambda_n) = 0 \tag{24}$$

One-Dimensional Rectangular Fin

The governing one-dimensional differential equation, with assumption of ℓ being very much less than the unit width in the z direction, is Eq. (25):

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} - 2Bi\theta = 0 \tag{25}$$

Two boundary conditions are given by Eqs. (26) and (27):

$$\theta|_{x=0} = 0 \tag{26}$$

$$\left.\frac{\mathrm{d}\theta}{\mathrm{d}x}\right|_{x=L} + Bi\theta|_{x=L} = 0 \tag{27}$$

By solving Eq. (25) with Eqs. (26) and (27), the temperature profile $\theta(x)$ can be written as Eq. (28):

$$\theta(x) = \frac{\sqrt{2}\cosh\left[\sqrt{2Bi}(L-x)\right] + \sqrt{Bi}\sinh\left[\sqrt{2Bi}(L-x)\right]}{\sqrt{2}\cosh\left(\sqrt{2Bi}L\right) + \sqrt{Bi}\sinh\left(\sqrt{2Bi}L\right)} \tag{28}$$

Dimensionless heat loss can be obtained by using Eq. (28) in Fourier's law and is expressed as Eq. (29):

$$Q_{1r} = \frac{q_{1r}}{k\theta_b} = \frac{2\sqrt{Bi}\sinh\left(\sqrt{2Bi}L\right) + \sqrt{2}Bi\cosh\left(\sqrt{2Bi}L\right)}{\sqrt{2}\cosh\left(\sqrt{2Bi}L\right) + \sqrt{Bi}\sinh\left(\sqrt{2Bi}L\right)} \tag{29}$$

Results

Isothermal profiles for $\theta = 0.7$, 0.8, and 0.9 within the traditional trapezoidal fin, whose tip height is a half of the base, for two different conditions (average of all Biot numbers is equal) are presented in Fig. 2. As expected, the temperature along the bottom surface decreases faster than that along the sloped top surface when the bottom Biot number is twice the value of the top Biot number. When the top Biot number is twice the value of bottom Biot number, the temperature along the sloped top surface decreases slightly faster than that along the bottom surface, but it appears approximately symmetric about the fin centerline. This appears to be a compromise between the thermal asymmetry and geometric asymmetry effects.

Relative errors in temperature at the tip of the rectangular fin calculated using the two-dimensional analytical method for $Bi = 0.2$ are listed in Table 1. It shows that relative error in the temperature between two- and one-dimensional cases increases up to almost 11% at $x = 10$ and $y = 0.5$ even though the absolute difference is small. With a view of temperature, when we substitute 0.99 as the value of ξ, the shape of the fin can be considered as a rectangular fin because the relative errors are less than 0.04%.

Table 2 lists errors relative to the heat loss based on Q_{2R}. In this table $Q_{\xi=0.999}$ means heat loss from an approximate retangular fin for $\xi = 0.999$ and $\alpha = \beta = 1$. The heat loss from a rectangular fin calculated using a one-dimensional analysis is within 1.6% relative to that calculated using two-dimensional analysis for the given range of Biot numbers and fin lengths. Kang and Look[13] showed that the relative errors of the heat loss between two- and one-dimensional analytical methods for a symmetric trapezoidal fin are within 1% until $L = 2$ and $Bi = 0.1$. In their study fin base height varied from $-\ell$ to ℓ, and fin tip height is fixed as half of base height. Also, for this range the relative error is less than 0.4% even though 0.99 is

Table 1 Relative error in the temperature at the fin tip for Bi = 0.2

		$[(\theta_{2r} - \theta_1)/\theta_{2r}]/i$, %		
y	L	$1r$	$\xi = 0.999$	$\xi = 0.99$
0	1	−2.829	−0.004	−0.034
	5	1.434	0	0.009
	10	6.265	0	−0.034
0.5	1	2.085	−0.005	−0.052
	5	6.164	0	0.009
	10	10.780	0	0

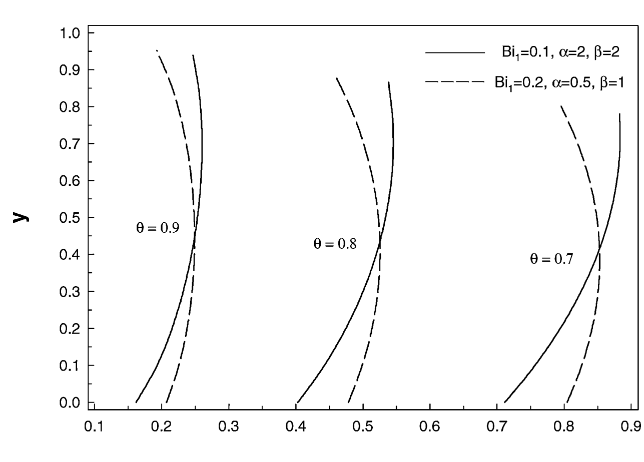

Fig. 2 Isothermal lines within a geometrically asymmetric trapezoidal fin: L = 2 and ξ = 0.5.

Table 2 Relative error in the heat loss

		$[(Q_{2r} - Q_i)/Q_{2r}]/i$, %		
L	Bi	$1r$	$\xi = 0.999$	$\xi = 0.99$
1	0.01	−0.102	0.027	0.318
	0.1	−0.870	0.025	0.264
	0.2	−1.553	0.023	0.217
5	0.01	−0.130	0	0.066
	0.1	−0.790	0	0.012
	0.2	−1.401	0	−0.003

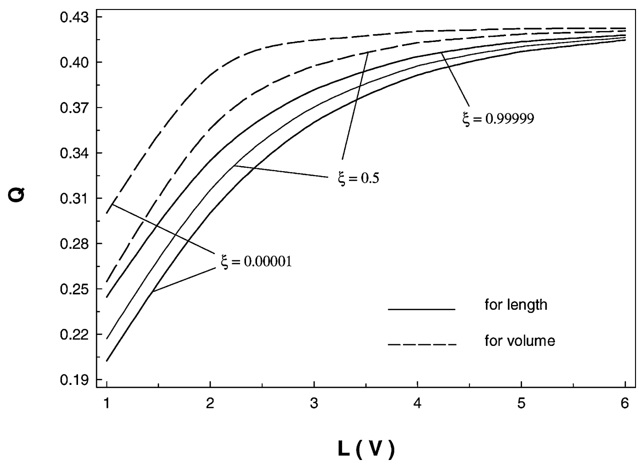

Fig. 3 Heat loss Q as a function of dimensionless fin length (volume) $L(V)$: Bi_1 = 0.1, α = 0.8, and β = 1.

Table 3 Ratio of the fin length [L^* for $(c \cdot Q_{max})/L^*$ for Q_{max}, %] with the variation of c in case of ξ = 0.99999, α = 0.5, and β = 1.0

	Bi_1			
$-c$	0.01	0.05	0.1	0.15
0.7	12.72	9.38	8.83	9.39
0.75	14.43	10.83	10.38	11.25
0.8	16.47	12.53	12.23	13.46
0.85	19.02	14.73	14.55	16.23
0.9	22.52	17.70	17.73	20.03
0.95	28.34	22.65	23.01	26.35
0.98	35.89	29.06	29.87	34.54
0.99	41.54	33.86	35.00	40.68

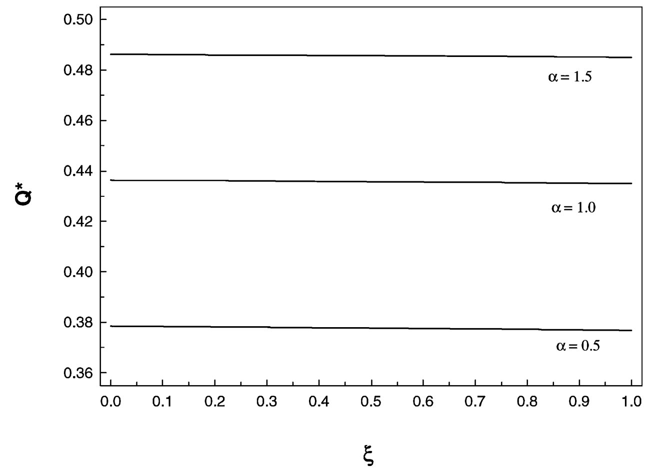

Fig. 4 Optimum heat loss Q^* vs ξ: Bi_1 = 0.1 and β = 1.

given as a value of ξ for an approximate rectangular fin. In the later optimization procedure, however, 0.99999 is used as the value of ξ to make the values of temperature and heat loss exactly the same as those obtained from a rectangular fin.

Figure 3 presents the heat loss from the fin as a function of the nondimensional fin length (and fin volume) for three values of ξ (0.00001 produces a triangular shape; 0.5 a traditional trapezoidal; and 0.99999 a rectangular shape). If we use 0 as a value of ξ, s becomes $1/L$, and the second term on the right-hand side in Eq. (5) vanishes. In this case Eq. (5) is still valid and becomes an energy balance equation for thermally and geometrically asymmetric triangular fins. If we use 1 as a value of ξ, then s becomes 0, and the first term on the right-hand side in Eq. (5) vanishes. In this case Eq. (5) becomes meaningless. For this reason we use $\xi = 0.99999$ for a rectangular fin in further discussion. Also we denote $\xi = 0.00001$ for a triangular fin even though $\xi = 0$ is possible because we actually use 0.00001 as a value of ξ for a triangular fin in this study. It has to be noticed that the defined dimensionless fin volume equals the dimensionless fin length for a rectangular fin. For L greater than 7, the shape of the fin is independent on the heat loss. In the case of L less than 7, the heat loss increases as ξ increases. These phenomena can be explained physically by noting that the increase in ξ means a larger extended surface is available for more heat loss. But this heat loss decreases as the fin length increases. For the same volume and fixed fin height the fin length for small values of ξ is longer than that for the large values of ξ. This increase of the heat loss with the decrease of ξ for the same volume appears to be contrary to that for the same fin length but is correct.

Table 3 lists the ratio of the fin length for a given fraction (i.e., c) of the maximum heat loss to the fin length for the maximum heat loss for an approximate rectangular fin with $\alpha = 0.5$ and $\beta = 1$. The dimensionless fin length for the maximum heat loss (i.e., L for Q_{max}) is 50.45 for $Bi_1 = 0.01$, 26.65 for $Bi_1 = 0.05$, and 17.70 for $Bi_1 = 0.15$. This table shows that the ratio of the fin length increases steadily as c increases, and it starts to increase rapidly after about $c = 0.98$ for given values of Bi_1. To get an increase of only 1%, from 0.98 to 0.99 for a value of c, the fin length ratio should be increased about 4.8–6.1% (e.g., from 35.89 to 41.54% for $Bi_1 = 0.01$ and from 29.87 to 35% for $Bi_1 = 0.1$). The ratio of the fin length must be increased about 60% for $Bi_1 = 0.01$ and 65% for $Bi_1 = 0.05$, 0.1 as the value of c increases from 0.99 to 1. So the heat loss equal to 98% of the maximum heat loss will be arbitrarily defined as the optimum heat loss in this study.

Though the fin shape factor ξ is usually based on fabricating convenience, it is worthwhile to explore its effect on the optimum designs. In Fig. 4 it is seen that, for optimum fins with fixed fin height, the effect of fin shape on the optimum heat loss is minimum and the optimum heat loss decreases only very slightly as the fin shape changes from triangular to rectangular. This is because the optimum heat loss is obtained for a fin long enough to make the effect of change in fin shape on the heat loss very small. It also can be noticed from Fig. 4 that the optimum heat loss increases about 11.46% as α increases from 1 to 1.5 while decreasing about 13.32% as α decreases from 1 to 0.5 for a typical trapezoidal fin. This phenomenon appears to be consistent because the increasing rate of heat loss becomes small as the overall average Biot number increases. As for the optimum dimensions, it is shown in Fig. 5 that the optimum fin length L^* remarkably decreases when ξ increases from 0 to 1. This means that if the fin height is fixed the optimum heat loss from triangular fins is always larger than the optimum heat loss from rectangular fins. However, as also presented in Fig. 5, the optimum volume V^* increases monotonically as ξ increases from 0 to 1. Figure 5 can be used to explain physically that the total triangular fin volume is always smaller than total rectangular fin volume even though the triangular fin length is longer than rectangular fin length for optimum heat loss.

Figure 6 presents the variation of the optimum heat loss Q^* as a function of α for a typical trapezoidal fin. Note that the optimum heat loss increases almost linearly as α increases for given fin top Biot numbers. The optimum dimensionless fin length (volume) vs α under the same condition given in Fig. 6 is shown in Fig. 7. The optimum fin length decreases somewhat rapidly at first and then decreases more slowly as α increases. It also can be assumed

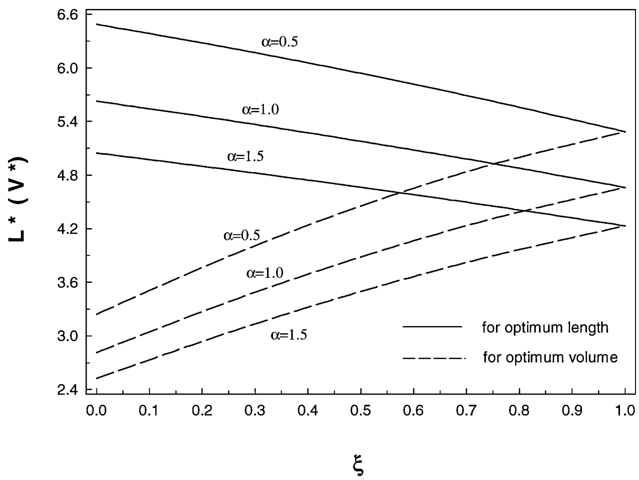

Fig. 5 Optimum dimensionless fin length (volume) L^* (V^*) vs ξ: Bi_1 = 0.1 and β = 1.

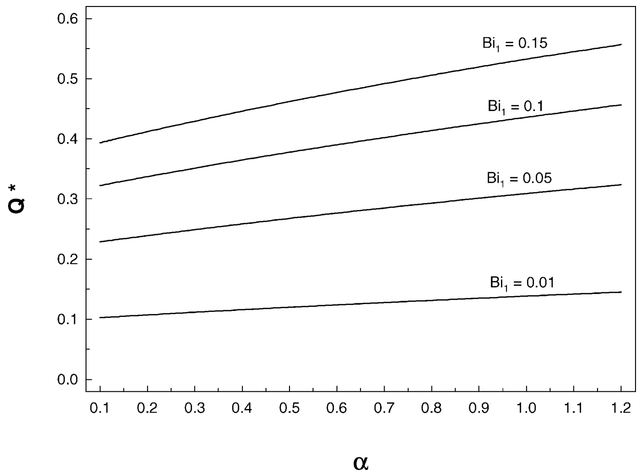

Fig. 6 Optimum heat loss Q^* as a function of α: ξ = 0.5 and β = 1.

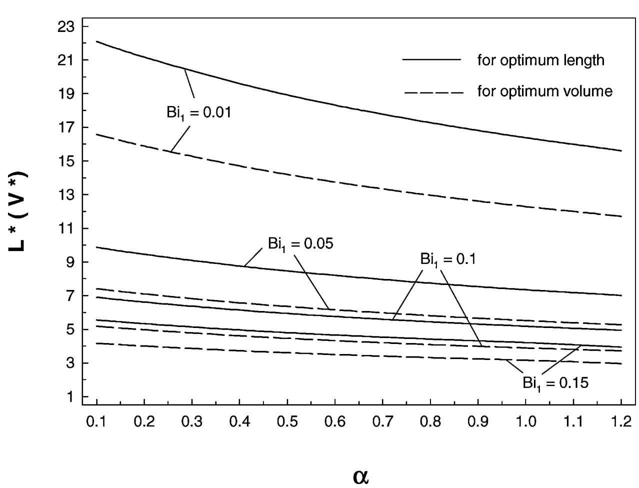

Fig. 7 Optimum dimensionless fin length (volume) L^* (V^*) as a function of α: ξ = 0.5 and β = 1.

from this figure that the optimum fin length deceases rapidly at first and then decreases more slowly as the fin top-surface Biot number increases for fixed values of α. The trend of the variation of fin volume is the same as that for fin length but becomes smaller because the shape factor ξ is fixed. (Fin volume is expressed as a function of L.)

The heat-transfer coefficient at the fin tip h_3 is generally larger than that of the other fin surfaces. The effects of the β ratio on

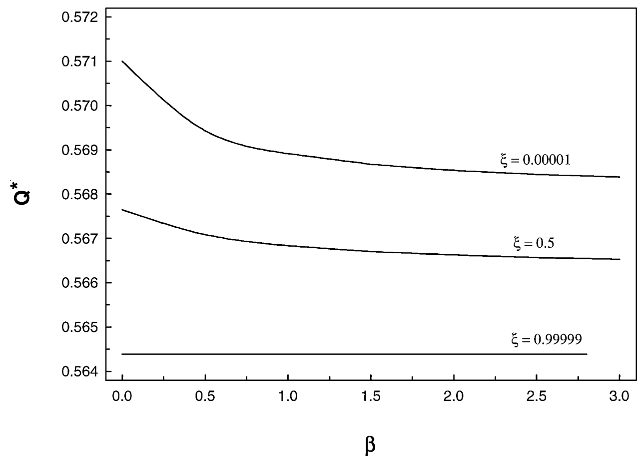

Fig. 8 Optimum heat loss Q^* vs β: Bi_1 = 0.2 and α = 0.7.

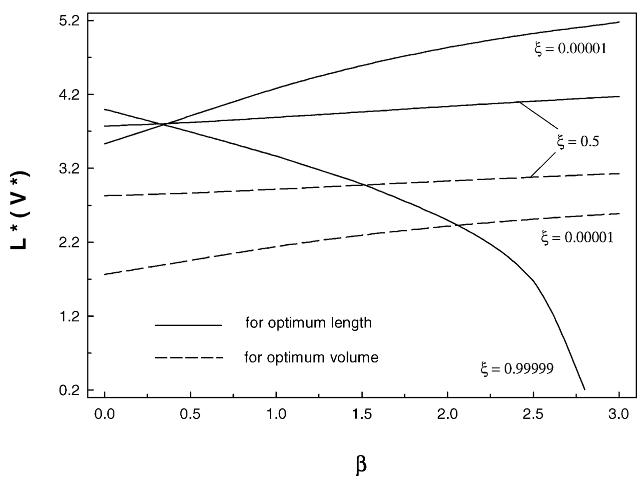

Fig. 9 Optimum dimensionless fin length (volume) L^* (V^*) vs β: Bi_1 = 0.2 and α = 0.7.

optimum designs are shown in Figs. 8 and 9. Figure 8 illustrates that the increase of β causes a decrease of Q^* for triangular and trapezoidal fins. This phenomenon can be explained by noting that the smaller fin tip convection coefficient restrains the heat flow in the fin tip direction and this restraint makes the heat flow to the top and bottom surfaces, which have larger convection coefficients. The variation of β has no effect on Q^* for a rectangular fin because the heat loss from the rectangular fin becomes maximum when the fin length is long enough so as not affected the fin tip condition. Figure 9 shows the optimum fin length (volume) L^* (V^*) corresponding to the optimum heat loss Q^* with the variation of β. The optimum fin length L^* increases as β increases for triangular and trapezoidal fins but not for a rectangular fin. Especially in the case of a rectangular fin, as shown in Figs. 8 and 9, β has no effect on Q^*, and hence $Q_{\max}$, but the fin length for Q^* (or $Q_{\max}$) decreases as β increases. When β is larger than 2.8, no optimum fin length exists because the maximum heat loss is the heat loss from the bare wall, (i.e., the fin is useless). The trend for corresponding optimum fin volume vs β (i.e., increasing for triangular and trapezoidal shapes while decreasing for the rectangular case) is the same, but the trends are somewhat different because of the shape factor.

Conclusions

In this study the optimum heat loss from the fin with fixed fin height at the base is selected arbitrarily as 98% of the maximum heat loss. It can be expected from Table 3 that the optimum fin length decreases slowly as the fraction of heat transfer to the maximum possible value (i.e., c) decreases from 0.98 and the optimum fin length increases rapidly as that fraction increases from 0.98. But the trend of the variation of the optimum dimensions are expected to be similar to the case of 98%, and conclusions based on this selection (i.e., the optimum heat loss as 98% of the maximum heat loss) have been drawn. The effects of fin shape and the ratio of fin tip-surface Biot number to fin top-surface Biot number (for $\beta > 1$) on the optimum heat loss are very small while the optimum heat loss increases linearly as the ratio of fin bottom-surface to top-surface Biot numbers increases. The optimum fin length decreases with the increase of the fin shape factor and the increase of the ratio of fin bottom-surface to fin top-surface Biot numbers when the other variables are fixed. Also, the optimum fin length is increased for triangular and trapezoidal fins while it is decreased for rectangular fins as the ratio of fin tip-surface to fin top-surface Biot numbers increases.

References

[1]Brown, A., "Optimum Dimensions of Uniform Annular Fins," *International Journal of Heat and Mass Transfer*, Vol. 8, No. 4, 1965, pp. 655–662.

[2]Hrymak, A. N., McRae, G. J., and Westerberg, A. W., "Combined Analysis and Optimization of Extended Heat Transfer Surfaces," *Journal of Heat Transfer*, Vol. 107, No. 3, 1985, pp. 527–532.

[3]Ullmann, A., and Kalman, H., "Efficiency and Optimized Dimensions of Annular Fin of Different Cross-Section Shapes," *International Journal of Heat and Mass Transfer*, Vol. 32, No. 6, 1989, pp. 1105–1110.

[4]Razelos, P., and Imre, K., "The Optimum Dimensions of Circular Fin with Variable Thermal Parameters," *Journal of Heat Transfer*, Vol. 102, No. 3, 1980, pp. 420–425.

[5]Gerencser, D. S., and Razan, A., "Optimization of Radiative-Convective Arrays of Pin Fins Including Mutual Irradiation Between Fins," *International Journal of Heat and Mass Transfer*, Vol. 38, No. 5, 1995, pp. 899–907.

[6]Casarosa, C., and Franco, A., "On the Optimum Thermal Design of Individual Longitudinal Fins with Rectangular Profile," *Heat Transfer Engineering*, Vol. 22, No. 1, 2001, pp. 51–71.

[7]Laor, K., and Kalman, H., "Performance and Optimum Dimensions of Different Cooling Fins with a Temperature-Dependent Heat Transfer Coefficient," *International Journal of Heat and Mass Transfer*, Vol. 39, No. 9, 1996, pp. 1993–2003.

[8]Look, D. C., Jr., "Fin on a Pipe (Insulated Tip): Minimum Conditions for Fin to be Beneficial," *Heat Transfer Engineering*, Vol. 16, No. 3, 1995, pp. 65–75.

[9]Kundu, B., and Das, P. K., "Performance Analysis and Optimization of Eccentric Annular Disk Fins," *Journal of Heat Transfer*, Vol. 121, No. 1, 1999, pp. 128–135.

[10]Look, D. C., and Kang, H. S., "Optimization of a Thermally Non-Symmetric Fin; Preliminary Evaluation," *International Journal of Heat and Mass Transfer*, Vol. 34, No. 4–5, 1992, pp. 2057–2060.

[11]Kraus, A. D., "Analysis of Extended Surface," *Journal of Heat Transfer*, Vol. 110, No. 4, 1988, pp. 1071–1073.

[12]Kang, H. S., and Look, D. C., "A Forced Analytical Scheme Applied to a Two Dimensional Fin: An Unsuccessful Venture," AIAA Paper 93-2854, July 1993.

[13]Kang, H. S., and Look, D. C., "A Comparison of Four Solution Methods for the Analysis of a Trapezoidal Fin," *Korean Society of Mechanical Engineers International Journal*, Vol. 13, No. 6, 1999, pp. 2057–2060.